

# Contents

	<b>A User's Guide to Optimal Transport .....</b>	<b>1</b>
	Luigi Ambrosio and Nicola Gigli	
1	Introduction .....	1
2	The Optimal Transport Problem .....	3
	2.1 Monge and Kantorovich Formulations of the Optimal Transport Problem .....	3
	2.2 Necessary and Sufficient Optimality Conditions .....	7
	2.3 The Dual Problem .....	13
	2.4 Existence of Optimal Maps .....	16
	2.5 Bibliographical Notes .....	26
3	The Wasserstein Distance $W_2$ .....	28
	3.1 $X$ Polish Space .....	29
	3.2 $X$ Geodesic Space .....	37
	3.3 $X$ Riemannian Manifold .....	47
	3.4 Bibliographical Notes .....	58
4	Gradient Flows .....	59
	4.1 Hilbertian Theory of Gradient Flows .....	59
	4.2 The Theory of Gradient Flows in a Metric Setting .....	61
	4.3 Applications to the Wasserstein Case .....	81
	4.4 Bibliographical Notes .....	92
5	Geometric and Functional Inequalities .....	93
	5.1 Brunn–Minkowski Inequality .....	94
	5.2 Isoperimetric Inequality .....	94
	5.3 Sobolev Inequality .....	95
	5.4 Bibliographical Notes .....	96
6	Variants of the Wasserstein Distance .....	97
	6.1 Branched Optimal Transportation .....	97
	6.2 Different Action Functional .....	99
	6.3 An Extension to Measures with Unequal Mass .....	100
	6.4 Bibliographical Notes .....	102

7	More on the Structure of $(\mathcal{P}_2(M), W_2)$ .....	103
	7.1 “Duality” Between the Wasserstein and the Arnold Manifolds .....	103
	7.2 On the Notion of Tangent Space .....	106
	7.3 Second Order Calculus .....	107
	7.4 Bibliographical Notes .....	130
8	Ricci Curvature Bounds .....	131
	8.1 Convergence of Metric Measure Spaces .....	134
	8.2 Weak Ricci Curvature Bounds: Definition and Properties .....	137
	8.3 Bibliographical Notes .....	150
	References .....	152
	<b>Hyperbolic Conservation Laws: An Illustrated Tutorial</b> .....	157
	Alberto Bressan	
1	Conservation Laws .....	158
	1.1 The Scalar Conservation Law .....	158
	1.2 Strictly Hyperbolic Systems .....	160
	1.3 Linear Systems .....	161
	1.4 Nonlinear Effects .....	163
	1.5 Loss of Regularity .....	164
	1.6 Wave Interactions .....	166
2	Weak Solutions .....	167
	2.1 Rankine–Hugoniot Conditions .....	168
	2.2 Construction of Shock Curves .....	172
	2.3 Admissibility Conditions .....	173
3	The Riemann Problem .....	179
	3.1 Some Examples .....	179
	3.2 A Class of Hyperbolic Systems .....	182
	3.3 Elementary Waves .....	184
	3.4 General Solution of the Riemann Problem .....	187
	3.5 The Riemann Problem for the p-System .....	190
	3.6 Error and Interaction Estimates .....	194
4	Global Solutions to the Cauchy Problem .....	196
	4.1 Front Tracking Approximations .....	197
	4.2 Bounds on the Total Variation .....	200
	4.3 Convergence to a Limit Solution .....	203
5	The Glimm Scheme .....	205
6	Continuous Dependence on the Initial Data .....	210
	6.1 Unique Solutions to the Scalar Conservation Law .....	211
	6.2 Linear Hyperbolic Systems .....	212
	6.3 Nonlinear Systems .....	213
7	Uniqueness of Solutions .....	216
	7.1 An Error Estimate for Front Tracking Approximations ...	217
	7.2 Characterization of Semigroup Trajectories .....	218
	7.3 Uniqueness Theorems .....	221

8	The Vanishing Viscosity Approach .....	223
	8.1 Local Decomposition by Traveling Waves .....	226
	8.2 Evolution of Gradient Components .....	230
	8.3 Lyapunov Functionals .....	231
	8.4 Continuous Dependence on the Initial Data .....	236
	8.5 The Semigroup of Vanishing Viscosity Limit Solutions...	237
9	Extensions and Open Problems .....	238
	9.1 Compactness Theorems .....	239
	9.2 An Elementary Error Estimate .....	240
	9.3 The Center Manifold Theorem .....	241
	References .....	243

## **Derivation of Non-local Macroscopic Traffic Equations and Consistent Traffic Pressures from Microscopic Car-Following Models ....**

Dirk Helbing		
1	Introduction .....	247
2	The Gradient Expansion Approach .....	248
3	The Linear Interpolation Approach .....	250
4	An Approach Reminding of Smooth Particle Hydrodynamics .....	253
	4.1 Derivation of the Continuity Equation .....	253
	4.2 Derivation of the Macroscopic Velocity Equation .....	255
	4.3 Discussion of the Non-locality .....	260
	4.4 Comparison with Other Macroscopic Traffic Models .....	260
5	Summary, Discussion, and Conclusions .....	266
	References .....	268

## **On the Controversy Around Daganzo's Requiem for and Aw–Rascle's Resurrection of Second-Order Traffic Flow Models .....**

Dirk Helbing and Anders Johansson		
1	Introduction .....	272
2	Summary of the Controversy Regarding Second-Order Traffic Flow Models .....	273
3	Linear Instability of Macroscopic Traffic Models .....	275
	3.1 Derivation of the Instability Condition .....	278
	3.2 Characteristic Speeds, Phase, and Group Velocities .....	279
4	Discussion .....	281
	4.1 Characteristic Speeds in the Aw–Rascle Model .....	281
	4.2 Payne's Traffic Model .....	282
	4.3 Characteristic Speeds Vs. Vehicle Speeds .....	284
5	Linear Instability and Characteristic Speeds of the Optimal Velocity Model .....	286
6	Summary, Conclusions, and Outlook .....	289
Appendix 1	Hyperbolic Sets of Partial Differential Equations and Characteristic Speeds .....	291
Appendix 2	Stability Analysis for Macroscopic Traffic Models .....	293

Appendix 3	Derivation of Formula (19) .....	294
Appendix 4	Meaning of the Group Velocity .....	296
Appendix 5	Linear Stability Analysis of the Optimal Velocity Model .....	297
Appendix 6	Correspondence of the Optimal Velocity Model with the Macroscopic Payne Model .....	299
References	.....	300
<b>Theoretical vs. Empirical Classification and Prediction of Congested Traffic States</b> .....		303
Dirk Helbing, Martin Treiber, Arne Kesting, and Martin Schönhof		
1	Introduction .....	303
2	On the Definition of Traffic Phases .....	306
3	Congested Traffic States .....	307
4	Derivation and Explanation of the Phase Diagram of Traffic States .....	309
	4.1 Transition to Congested Traffic for Small Bottlenecks ....	313
	4.2 Conditions for Different Kinds of Congested Traffic After the Breakdown of Traffic Flow .....	315
5	Combinations of On- and Off-Ramps .....	318
6	Other Phase Diagrams and Universality Classes of Models ....	321
7	Empirical Phase Diagram .....	324
	7.1 Reply to Criticisms of Phase Diagrams for Traffic Models with a Fundamental Diagram .....	325
	7.2 On the Validity of Traffic Models .....	326
8	Summary, Conclusions, and Outlook .....	328
Appendix 1	Modeling of Source and Sink Terms (In- and Outflows) .....	329
Appendix 2	Parameter Dependence of the Instability Thresholds in the Intelligent Driver Model .....	330
References	.....	331
<b>Self-Organized Network Flows</b> .....		335
Dirk Helbing, Jan Siegmeier, and Stefan Lämmer		
1	Introduction .....	335
2	Flows in Networks .....	336
	2.1 Flow Conservation Laws .....	337
	2.2 Two Views on Traffic Jams .....	339
3	Treatment of Merging, Diverging and Intersection Points .....	344
	3.1 Diverging Flows: One Inflow and Several Outflows .....	345
	3.2 Merging Flows: Two Inflows and One Outflow .....	345
	3.3 A Side Road Merging with a Main Road .....	346
	3.4 Intersection-Free Designs of Road Networks .....	347
	3.5 Two Inflows and Two Outflows .....	348
	3.6 Inefficiencies Due to Coordination Problems .....	350
4	Towards a Self-Organized Traffic Light Control .....	351
5	Summary and Outlook .....	353
References	.....	354

<b>Operation Regimes and Slower-is-Faster-Effect in the Control of Traffic Intersections</b> .....		357
Dirk Helbing and Amin Mazlounian		
1	Introduction .....	357
	1.1 Paradoxical Behavior of Transport Systems .....	358
2	Specification of the Traffic System Under Consideration .....	359
3	Consideration of Traffic Flows .....	362
4	Travel-Time-Oriented Signal Operation .....	364
	4.1 The Optimize-One-Phase Approach .....	365
	4.2 Transformation to Dimensionless Variables and Parameters .....	368
	4.3 Control Strategies and Slower-is-Faster Effect .....	371
	4.4 Operation Regimes for Periodic Operation .....	372
	4.5 Minimization of Vehicle Queues .....	375
	4.6 Complexity of Traffic Light Control .....	375
5	Optimize-Multiple-Phases Approach .....	376
	5.1 Combined Flow-and-Delay Time Optimization .....	377
6	Summary, Discussion, and Outlook .....	383
	6.1 Self-Organized Traffic Light Control .....	385
Appendix 1	Considering the Price of Stopping Vehicles .....	387
Appendix 2	More Than Two Traffic Phases .....	389
Appendix 3	Limited Forecast Time Horizon .....	391
References	.....	392
<b>Modeling and Optimization of Scalar Flows on Networks</b> .....		395
Simone Göttlich and Axel Klar		
1	Introduction .....	395
2	Traffic Flow Networks .....	397
	2.1 Network Models Based on Scalar Partial Differential Equations .....	397
	2.2 Simplified Dynamics on the Network .....	410
	2.3 Optimization .....	415
	2.4 Summary .....	436
3	Modeling Supply Networks .....	437
	3.1 Network Models Based on Scalar Conservation Laws ....	437
	3.2 Optimization Problems .....	442
	3.3 Numerical Results .....	451
	3.4 Summary .....	459
References	.....	459
<b>Control and Stabilization of Waves on 1-d Networks</b> .....		463
Enrique Zuazua		
1	Introduction and Main Results .....	464
2	The Wave Equation on a Network .....	468
3	Main Results on Observability and Controllability .....	474
	3.1 Summary of Known Results .....	474
	3.2 The Weighted Observability Inequality .....	477

4	Stabilization .....	478
	4.1 Problem Formulation .....	478
	4.2 Observability for the Damped System.....	482
	4.3 The Interpolation Inequality .....	484
	4.4 The Main Result .....	486
5	Further Comments and Open Problems .....	487
References	.....	491