

Contents

1	Preliminaries	1
1.1	Sobolev Spaces and Embedding Theorems	1
1.2	Critical Point	5
1.3	Cone and Partial Order	11
1.4	Brouwer Degree	12
1.5	Compact Map and Leray–Schauder Degree	13
1.5.1	Definitions	13
1.5.2	Properties of Compact Maps	14
1.5.3	The Leray–Schauder Degree	15
1.6	Fredholm Operators	17
1.7	Fixed Point Index	18
1.8	Banach’s Contract Theorem, Implicit Functions Theorem	20
1.9	Krein–Rutman Theorem	20
1.10	Bifurcation Theory	21
1.11	Rearrangements of Sets and Functions	23
1.12	Genus and Category	25
1.13	Maximum Principles and Symmetry of Solution	27
1.14	Comparison Theorems	28
2	Cone and Partial Order Methods	35
2.1	Increasing Operators	35
2.2	Decreasing Operators	45
2.3	Mixed Monotone Operators	54
2.4	Applications of Mixed Monotone Operators	74
2.5	Further Results on Cones and Partial Order Methods	84
3	Minimax Methods	99
3.1	Mountain Pass Theorem and Minimax Principle	99
3.2	Linking Methods	101
3.3	Local Linking Methods	103
3.3.1	Deformation Lemmas	104

3.3.2	The Three Critical Points Theorem for Functionals Bounded Below	107
3.3.3	Super-quadratic Functionals	110
3.3.4	Asymptotically Quadratic Functionals	112
3.3.5	Applications to Elliptic Boundary Value Problems	116
3.3.6	Local Linking and Critical Groups	121
4	Bifurcation and Critical Point	131
4.1	Introduction	131
4.2	Main Results with Parameter	133
4.3	Equations Without the Parameter	141
5	Solutions of a Class of Monge–Ampère Equations	143
5.1	Introduction	143
5.2	Moving Plane Argument	145
5.3	Existence and Non-existence Results	149
5.4	Bifurcation and the Equation with a Parameter	153
5.5	Appendix	171
6	Topological Methods and Applications	175
6.1	Superlinear System of Integral Equations and Applications	175
6.1.1	Introduction	175
6.1.2	Existence of Non-trivial Solutions	175
6.1.3	Application to Two-Point Boundary Value Problems	185
6.2	Existence of Positive Solutions for a Semilinear Elliptic System	186
6.2.1	Introduction	186
6.2.2	Existence of Positive Solutions	189
7	Dancer–Fučík Spectrum	199
7.1	The Spectrum of a Self-adjoint Operator	199
7.2	Dancer–Fučík Spectrum on Bounded Domains	200
7.3	Dancer–Fučík Point Spectrum on \mathbb{R}^N	204
7.3.1	Introduction	204
7.3.2	The Trivial Part of the Fučík Point Spectrum	205
7.3.3	Non-trivial Fučík Eigenvalues by Minimax Methods	208
7.3.4	Some Properties of the First Curve and the Corresponding Eigenfunctions	212
7.4	Dancer–Fučík Spectrum and Asymptotically Linear Elliptic Problems	215
7.4.1	Introduction	215
7.4.2	Proofs of Main Theorems	217
8	Sign-Changing Solutions	221
8.1	Sign-Changing Solutions for Superlinear Dirichlet Problems	221
8.1.1	Nehari Manifold and Sign-Changing Solutions	221
8.1.2	Additional Properties of Sign-Changing Solutions to Superlinear Elliptic Equations	226

8.2	Sign-Changing Solutions for Jumping Nonlinear Problems	231
8.2.1	On Limit Equation of Lotka–Volterra Competing System with Two Species	231
8.2.2	On General Jumping Nonlinear Problems	235
8.2.3	Sign-Changing Solutions of p -Laplacian Equations	244
8.2.4	Sign-Changing Solutions of Schrödinger Equations	246
9	Extension of Brezis–Nirenberg’s Results and Quasilinear Problems	249
9.1	Introduction	249
9.2	$W_0^{1,p}(\Omega)$ Versus $C_0^1(\bar{\Omega})$ Local Minimizers	251
9.3	Multiplicity Results for the Quasilinear Problems	253
9.4	Uniqueness Results	267
10	Nonlocal Kirchhoff Elliptic Problems	271
10.1	Introduction	271
10.2	Yang Index and Critical Groups to Nonlocal Problems	272
10.3	Variational Methods and Invariant Sets of Descent Flow	278
10.4	Uniqueness of Solution for a Class of Kirchhoff-Type Equations	282
11	Free Boundary Problems, System of Equations for Bose–Einstein Condensate and Competing Species	285
11.1	Competing System with Many Species	285
11.1.1	Existence and Uniqueness of Positive Solution	285
11.1.2	The Limit Spatial Segregation System of Competing Systems	290
11.2	Optimal Partition Problems	291
11.2.1	An Optimal Partition Problem Related to Nonlinear Eigenvalues	291
11.2.2	An Optimal Partition Problem for Eigenvalues	295
11.3	Schrödinger Systems from Bose–Einstein Condensate	298
11.3.1	Existence of Solutions for Schrödinger Systems	300
11.3.2	The Limit State of Schrödinger Systems	310
11.3.3	C^α Estimate of the Solutions of Parabolic Systems	316
References	319
Index	329