Contents

Part I Principles of Monte Carlo Methods

1	Intr	oductio	m	3
	1.1	Why U	Jse Probabilistic Models and Simulations?	3
		1.1. 1	What Are the Reasons for Probabilistic Models?	4
		1.1.2	What Are the Objectives of Random Simulations?	6
	1.2	Organ	ization of the Monograph	9
2	Stro	ng Law	v of Large Numbers and Monte Carlo Methods	13
	2.1	Strong	g Law of Large Numbers, Examples of Monte Carlo Methods	13
		2.1.1	Strong Law of Large Numbers, Almost Sure Convergence .	13
		2.1.2	Buffon's Needle	15
		2.1.3	Neutron Transport Simulations	15
		2.1.4	Stochastic Numerical Methods for Partial Differential	
			Equations	17
	2.2	Simula	ation Algorithms for Simple Probability Distributions	18
		2.2.1	Uniform Distributions	19
		2.2.2	Discrete Distributions	20
		2.2.3	Gaussian Distributions	21
		2.2.4	Cumulative Distribution Function Inversion, Exponential	
			Distributions	22
		2.2.5	Rejection Method	23
			ete-Time Martingales, Proof of the SLLN	25
		2.3.1	Reminders on Conditional Expectation	25
		2.3.2	Martingales and Sub-martingales, Backward Martingales	27
		2.3.3	Proof of the Strong Law of Large Numbers	30
	2.4	Proble	ems	33
3	Non	-asymp	ototic Error Estimates for Monte Carlo Methods	37
	3.1	Conve	ergence in Law and Characteristic Functions	37
	3.2		al Limit Theorem	40
		3.2.1	Asymptotic Confidence Intervals	41
	3.3	Berry-	-Esseen's Theorem	42





	3.4	Bikelis' Theorem	45		
		3.4.1 Absolute Confidence Intervals	45		
	3.5	Concentration Inequalities	47		
		3.5.1 Logarithmic Sobolev Inequalities	48		
		3.5.2 Concentration Inequalities, Absolute Confidence Intervals .	50		
	3.6	Elementary Variance Reduction Techniques	54		
		3.6.1 Control Variate	54		
		3.6.2 Importance Sampling	55		
	3.7	Problems	60		
Part II Exact and Approximate Simulation of Markov Processes					
4	Pois	son Processes as Particular Markov Processes	67		
	4.1	Quick Introduction to Markov Processes	67		
		4.1.1 Some Issues in Markovian Modeling	67		
		4.1.2 Rudiments on Processes, Sample Paths, and Laws	68		
	4.2	Poisson Processes: Characterization, Properties	69		
		4.2.1 Point Processes and Poisson Processes	69		
		4.2.2 Simple and Strong Markov Property	75		
		4.2.3 Superposition and Decomposition	77		
	4.3	Simulation and Approximation	80		
	ч.5	4.3.1 Simulation of Inter-arrivals	80		
		4.3.2 Simulation of Independent Poisson Processes	81		
		-	82		
	4.4		82 85		
	4.4	Problems			
5		crete-Space Markov Processes	89		
	5.1	Characterization, Specification, Properties	89		
		5.1.1 Measures, Functions, and Transition Matrices	89		
		5.1.2 Simple and Strong Markov Property	91		
		5.1.3 Semigroup, Infinitesimal Generator, and Evolution Law	95		
	5.2	Constructions, Existence, Simulation, Equations	99		
		5.2.1 Fundamental Constructions	99		
		5.2.2 Explosion or Existence for a Markov Process	101		
		5.2.3 Fundamental Simulation, Fictitious Jump Method	103		
		5.2.4 Kolmogorov Equations, Feynman–Kac Formula	105		
		5.2.5 Generators and Semigroups in Bounded Operator Algebras	107		
		5.2.6 A Few Case Studies	112		
	5.3	Problems			
6	Con	tinuous-Space Markov Processes with Jumps	121		
	6.1	Preliminaries			
		6.1.1 Measures, Functions, and Transition Kernels			
		6.1.2 Markov Property, Finite-Dimensional Marginals			
		6.1.3 Semigroup, Infinitesimal Generator			
	6.2	Markov Processes Evolving Only by Isolated Jumps			
		6.2.1 Semigroup, Infinitesimal Generator, and Evolution Law			

Contents

		6.2.2 Construction, Simulation, Existence	30
		6.2.3 Kolmogorov Equations, Feynman–Kac Formula, Bounded	
		Generator Case	33
	6.3	Markov Processes Following an Ordinary Differential Equation	
			36
		6.3.1 Sample Paths, Evolution, Integro-Differential Generator 1	36
		6.3.2 Construction, Simulation, Existence	41
		6.3.3 Kolmogorov Equations, Feynman–Kac Formula 1	44
		6.3.4 Application to Kinetic Equations	46
		6.3.5 Further Extensions	49
	6.4	Problems	51
7	Disc	retization of Stochastic Differential Equations	55
	7.1	Reminders on Itô's Stochastic Calculus	55
		7.1.1 Stochastic Integrals and Itô Processes	55
		7.1.2 Itô's Formula, Existence and Uniqueness of Solutions	
		of Stochastic Differential Equations	60
		7.1.3 Markov Properties, Martingale Problems and Fokker-	
		Planck Equations	62
	7.2	Euler and Milstein Schemes	65
	7.3	Moments of the Solution and of Its Approximations 1	
	7.4		73
	7.5	Monte Carlo Methods for Parabolic Partial Differential Equations . 1	
		7.5.1 The Principle of the Method	
		7.5.2 Introduction of the Error Analysis	
	7.6	Optimal Convergence Rate: The Talay–Tubaro Expansion 1	
	7.7		85
	7.8	Probabilistic Interpretation and Estimates for Parabolic Partial	
			86
	7.9	Problems	91
Part	t III	Variance Reduction, Girsanov's Theorem, and Stochastic	
		orithms	
8	Vari	ance Reduction and Stochastic Differential Equations	99
U U	8.1	Preliminary Reminders on the Girsanov Theorem	
	8.2	Control Variates Method	
	8.3	Variance Reduction for Sensitivity Analysis	
		8.3.1 Differentiable Terminal Conditions	
			204
	8.4		206
	8.5	· · · ·	209
	8.6	Problems	210
9	Stoc	hastic Algorithms	213
	9.1	Introduction	
	9.2	Study in an Idealized Framework	

	9.2.1 Definitions
	9.2.2 The Ordinary Differential Equation Method, Martingale
	Increments
	9.2.3 Long-Time Behavior of the Algorithm
9.3	Variance Reduction for Monte Carlo Methods
	9.3.1 Searching for an Importance Sampling
	9.3.2 Variance Reduction and Stochastic Algorithms
9.4	Problems
Appendiz	x Solutions to Selected Problems
Referenc	es
Index .	