

## Contents

**Foreword to the First Edition** XVII

**Preface to the First Edition** XIX

**Preface to the Second Edition** XXIII

**Acknowledgements** XXV

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Classification of Rotor Systems	1
1.2	Historical Perspective	3
	References	8
<b>2</b>	<b>Vibrations of Massless Shafts with Rigid Disks</b>	<b>11</b>
2.1	General Considerations	11
2.2	Rotor Unbalance	11
2.3	Lateral Vibrations of an Elastic Shaft with a Disk at Its Center	13
2.3.1	Derivation of Equations of Motion	13
2.3.2	Free Vibrations of an Undamped System and Whirling Modes	14
2.3.3	Synchronous Whirl of an Undamped System	16
2.3.4	Synchronous Whirl of a Damped System	20
2.3.5	Energy Balance	22
2.4	Inclination Vibrations of an Elastic Shaft with a Disk at Its Center	23
2.4.1	Rotational Equations of Motion for Single Axis Rotation	23
2.4.2	Equations of Motion	23
2.4.3	Free Vibrations and Natural Angular Frequency	27
2.4.4	Gyroscopic Moment	29
2.4.5	Synchronous Whirl	33
2.5	Vibrations of a 4 DOF System	34
2.5.1	Equations of Motion	34
2.5.1.1	Derivation by Using the Results of 2 DOF System	35
2.5.1.2	Derivation by Lagrange's Equations	37
2.5.2	Free Vibrations and a Natural Frequency Diagram	40
2.5.3	Synchronous Whirling Response	42
2.6	Vibrations of a Rigid Rotor	43
2.6.1	Equations of Motion	43

2.6.2	Free Whirling Motion and Whirling Modes	45
2.7	Approximate Formulas for Critical Speeds of a Shaft with Several Disks	46
2.7.1	Rayleigh's Method	47
2.7.2	Dunkerley's Formula	48
	References	48
<b>3</b>	<b>Vibrations of a Continuous Rotor</b>	<b>49</b>
3.1	General Considerations	49
3.2	Equations of Motion	50
3.3	Free Whirling Motions and Critical Speeds	55
3.3.1	Analysis Considering Only Transverse Motion	56
3.3.2	Analysis Considering the Gyroscopic Moment and Rotary Inertia	58
3.3.3	Major Critical Speeds	59
3.4	Synchronous Whirl	60
	References	65
<b>4</b>	<b>Balancing</b>	<b>67</b>
4.1	Introduction	67
4.2	Classification of Rotors	67
4.3	Balancing of a Rigid Rotor	69
4.3.1	Principle of Balancing	69
4.3.1.1	Two-Plane Balancing	69
4.3.1.2	Single-Plane Balancing	70
4.3.2	Balancing Machine	71
4.3.2.1	Static Balancing Machine	71
4.3.2.2	Dynamic Balancing Machine	71
4.3.3	Field Balancing	75
4.3.4	Various Expressions of Unbalance	77
4.3.4.1	Resultant Unbalance $U$ and Resultant Unbalance Moment $V$	77
4.3.4.2	Dynamic Unbalance ( $U_1, U_2$ )	79
4.3.4.3	Static Unbalance $U$ and Couple Unbalance [ $U_c, -U_c$ ]	80
4.3.5	Balance Quality Grade of a Rigid Rotor	82
4.3.5.1	Balance Quality Grade	82
4.3.5.2	How to Use the Standards	84
4.4	Balancing of a Flexible Rotor	86
4.4.1	Effect of the Elastic Deformation of a Rotor	86
4.4.2	Modal Balancing Method	87
4.4.2.1	N-Plane Modal Balancing	88
4.4.2.2	( $N + 2$ )-Plane Modal Balancing	90
4.4.3	Influence Coefficient Method	90
	References	92
<b>5</b>	<b>Vibrations of an Asymmetrical Shaft and an Asymmetrical Rotor</b>	<b>93</b>
5.1	General Considerations	93

5.2	Asymmetrical Shaft with a Disk at Midspan	94
5.2.1	Equations of Motion	94
5.2.2	Free Vibrations and Natural Frequency Diagrams	95
5.2.2.1	Solutions in the Ranges $\omega > \omega_{c1}$ and $\omega < \omega_{c2}$	98
5.2.2.2	Solutions in the Range $\omega_{c1} > \omega > \omega_{c2}$	99
5.2.3	Synchronous Whirl in the Vicinity of the Major Critical Speed	100
5.3	Inclination Motion of an Asymmetrical Rotor Mounted on a Symmetrical Shaft	102
5.3.1	Equations of Motion	103
5.3.2	Free Vibrations and a Natural Frequency Diagram	108
5.3.3	Synchronous Whirl in the Vicinity of the Major Critical Speed	109
5.4	Double-Frequency Vibrations of an Asymmetrical Horizontal Shaft	110
	References	113
<b>6</b>	<b>Nonlinear Vibrations</b>	<b>115</b>
6.1	General Considerations	115
6.2	Causes and Expressions of Nonlinear Spring Characteristics: Weak Nonlinearity	115
6.3	Expressions of Equations of Motion Using Physical and Normal Coordinates	121
6.4	Various Types of Nonlinear Resonances	123
6.4.1	Harmonic Resonance	124
6.4.1.1	Solution by the Harmonic Balance Method	124
6.4.1.2	Solution Using Normal Coordinates	128
6.4.2	Subharmonic Resonance of Order 1/2 of a Forward Whirling Mode	130
6.4.3	Subharmonic Resonance of Order 1/3 of a Forward Whirling Mode	132
6.4.4	Combination Resonance	133
6.4.5	Summary of Nonlinear Resonances	136
6.5	Nonlinear Resonances in a System with Radial Clearance: Strong Nonlinearity	139
6.5.1	Equations of Motion	141
6.5.2	Harmonic Resonance and Subharmonic Resonances	142
6.5.3	Chaotic Vibrations	144
6.6	Nonlinear Resonances of a Continuous Rotor	145
6.6.1	Representations of Nonlinear Spring Characteristics and Equations of Motion	146
6.6.2	Transformation to Ordinary Differential Equations	149
6.6.3	Harmonic Resonance	150
6.6.4	Summary of Nonlinear Resonances	151
6.7	Internal Resonance Phenomenon	152
6.7.1	Examples of the Internal Resonance Phenomenon	152
6.7.2	Subharmonic Resonance of Order 1/2	153

6.7.3	Chaotic Vibrations in the Vicinity of the Major Critical Speed	156
	References	158
<b>7</b>	<b>Self-Excited Vibrations due to Internal Damping</b>	<b>161</b>
7.1	General Considerations	161
7.2	Friction in Rotor Systems and Its Expressions	161
7.2.1	External Damping	162
7.2.2	Hysteretic Internal Damping	162
7.2.3	Structural Internal Damping	167
7.3	Self-Excited Vibrations due to Hysteretic Damping	168
7.3.1	System with Linear Internal Damping Force	169
7.3.2	System with Nonlinear Internal Damping Force	171
7.4	Self-Excited Vibrations due to Structural Damping	173
	References	176
<b>8</b>	<b>Nonstationary Vibrations during Passage through Critical Speeds</b>	<b>177</b>
8.1	General Considerations	177
8.2	Equations of Motion for Lateral Motion	178
8.3	Transition with Constant Acceleration	179
8.4	Transition with Limited Driving Torque	183
8.4.1	Characteristics of Power Sources	183
8.4.2	Steady-State Vibration	184
8.4.3	Stability Analysis	187
8.4.4	Nonstationary Vibration	188
8.5	Analysis by the Asymptotic Method (Nonlinear System, Constant Acceleration)	189
8.5.1	Equations of Motion and Their Transformation to a Normal Coordinate Expression	190
8.5.2	Steady-State Solution	192
8.5.3	Nonstationary Vibration	194
	References	196
<b>9</b>	<b>Vibrations due to Mechanical Elements</b>	<b>199</b>
9.1	General Considerations	199
9.2	Ball Bearings	199
9.2.1	Vibration and Noise in Rolling-Element Bearings	199
9.2.1.1	Vibrations due to the Passage of Rolling Elements	200
9.2.1.2	Natural Vibrations of Outer Rings	202
9.2.1.3	Geometrical Imperfection	204
9.2.1.4	Other Noises	205
9.2.2	Resonances of a Rotor Supported by Rolling-Element Bearings	205
9.2.2.1	Resonances due to Shaft Eccentricity	205
9.2.2.2	Resonances due to the Directional Difference in Stiffness	206
9.2.2.3	Vibrations of a Horizontal Rotor due to the Passage of Rolling Elements	208

9.2.2.4	Vibrations due to the Coexistence of the Passage of Rolling Elements and a Shaft Initial Bend	208
9.3	Bearing Pedestals with Directional Difference in Stiffness	209
9.4	Universal Joint	211
9.5	Rubbing	215
9.5.1	Equations of Motion	217
9.5.2	Numerical Simulation	218
9.5.3	Theoretical Analysis	220
9.5.3.1	Forward Rubbing	220
9.5.3.2	Backward Rubbing	221
9.6	Self-Excited Oscillation in a System with a Clearance between Bearing and Housing	222
9.6.1	Experimental Setup and Experimental Results	223
9.6.2	Analytical Model and Reduction of Equations of Motion	224
9.6.3	Numerical Simulation	226
9.6.4	Self-Excited Oscillations	227
9.6.4.1	Analytical Model and Equations of Motion	227
9.6.4.2	Stability of a Synchronous Whirl	228
9.6.4.3	Mechanism of a Self-Excited Oscillation	229
	References	232
<b>10</b>	<b>Flow-Induced Vibrations</b>	<b>235</b>
10.1	General Considerations	235
10.2	Oil Whip and Oil Whirl	235
10.2.1	Journal Bearings and Self-Excited Vibrations	236
10.2.2	Reynolds Equation	239
10.2.3	Oil Film Force	240
10.2.3.1	Short Bearing Approximation	241
10.2.3.2	Long Bearing Approximation	243
10.2.4	Stability Analysis of an Elastic Rotor	243
10.2.5	Oil Whip Prevention	246
10.3	Seals	248
10.3.1	Plain Annular Seal	248
10.3.2	Labyrinth Seal	251
10.4	Tip Clearance Excitation	251
10.5	Hollow Rotor Partially Filled with Liquid	252
10.5.1	Equations Governing Fluid Motion and Fluid Force	254
10.5.2	Asynchronous Self-Excited Whirling Motion	256
10.5.3	Resonance Curves at the Major Critical Speed (Synchronous Oscillation)	257
	References	261
<b>11</b>	<b>Vibration Suppression</b>	<b>263</b>
11.1	Introduction	263
11.2	Vibration Absorbing Rubber	263

11.3	Theory of Dynamic Vibration Absorber	263
11.4	Squeeze-Film Damper Bearing	264
11.5	Ball Balancer	266
11.5.1	Fundamental Characteristics and the Problems	266
11.5.2	Countermeasures to the Problems	268
11.6	Discontinuous Spring Characteristics	271
11.6.1	Fundamental Characteristics and the Problems	271
11.6.2	Countermeasures to the Problems	273
11.6.3	Suppression of Unstable Oscillations of an Asymmetrical Shaft	274
11.7	Leaf Spring	276
11.8	Viscous Damper	277
11.9	Suppression of Rubbing	278
	References	280
<b>12</b>	<b>Some Practical Rotor Systems</b>	<b>283</b>
12.1	General Consideration	283
12.2	Steam Turbines	283
12.2.1	Construction of a Steam Turbine	283
12.2.2	Vibration Problems of a Steam Turbine	286
12.2.2.1	Poor Accuracy in the Manufacturing of Couplings	286
12.2.2.2	Thermal Bow	287
12.2.2.3	Vibrations of Turbine Blades	287
12.2.2.4	Oil Whip and Oil Whirl	290
12.2.2.5	Labyrinth Seal	290
12.2.2.6	Steam Whirl	290
12.3	Wind Turbines	290
12.3.1	Structure of a Wind Turbine	290
12.3.2	Campbell Diagram of a Wind Turbine with Two Teetered Blades	292
12.3.3	Excitation Forces in Wind Turbines	294
12.3.4	Example: Steady-State Oscillations of a Teetered Two-Bladed Wind Turbine	295
12.3.4.1	Wind Velocity	296
12.3.4.2	Vibration of the Tower	296
12.3.4.3	Flapwise Bending Vibration of the Blade	297
12.3.4.4	Chordwise Bending Vibration of the Blade	297
12.3.4.5	Torque Variation of the Low-Speed Shaft	297
12.3.4.6	Variation of the Teeter Angle	297
12.3.4.7	Variation of the Pitch Angle	297
12.3.4.8	Gear	297
12.3.5	Balancing of a Rotor	298
12.3.6	Vibration Analysis of a Blade Rotating in a Vertical Plane	299
12.3.6.1	Derivation of Equations of Motion	299
12.3.6.2	Natural Frequencies	302
12.3.6.3	Forced Oscillation	302

12.3.6.4	Parametrically Excited Oscillation	303
	References	305
<b>13</b>	<b>Cracked Rotors</b>	<b>307</b>
13.1	General Considerations	307
13.2	Modeling and Equations of Motion	309
13.2.1	Piecewise Linear Model (PWL Model)	309
13.2.2	Power Series Model (PS Model)	311
13.3	Numerical Simulation (PWL Model)	312
13.3.1	Horizontal Rotor	312
13.3.2	Vertical Rotor	313
13.4	Theoretical Analysis (PS Model)	313
13.4.1	Forward Harmonic Resonance $[+\omega]$ (Horizontal Rotor)	313
13.4.2	Forward Harmonic Resonance $[+\omega]$ (Vertical Rotor)	315
13.4.3	Forward Superharmonic Resonance $[+2\omega]$ (Horizontal Rotor)	315
13.4.4	Other Kinds of Resonance	317
13.4.4.1	Backward Harmonic Resonance $[-\omega]$	317
13.4.4.2	Forward Superharmonic Resonance $[+3\omega]$	317
13.4.4.3	Forward Subharmonic Resonance $[+(1/2)\omega]$	318
13.4.4.4	Forward Super-Subharmonic Resonance $[+(3/2)\omega]$	319
13.4.4.5	Combination Resonance	320
13.5	Case History in Industrial Machinery	321
	References	324
<b>14</b>	<b>Finite Element Method</b>	<b>327</b>
14.1	General Considerations	327
14.2	Fundamental Procedure of the Finite Element Method	327
14.3	Discretization of a Rotor System	328
14.3.1	Rotor Model and Coordinate Systems	328
14.3.2	Equations of Motion of an Element	329
14.3.2.1	Rigid Disk	329
14.3.2.2	Finite Rotor Element	330
14.3.3	Equations of Motion for a Complete System	336
14.3.3.1	Model I: (Uniform Elastic Rotor)	336
14.3.3.2	Model II: Disk–Shaft System	340
14.3.3.3	Variation of Equations of Motion	343
14.4	Free Vibrations: Eigenvalue Problem	345
14.5	Forced Vibrations	347
14.6	Alternative Procedure	349
	References	350
<b>15</b>	<b>Transfer Matrix Method</b>	<b>351</b>
15.1	General Considerations	351
15.2	Fundamental Procedure of the Transfer Matrix Method	351
15.2.1	Analysis of Free Vibration	351

15.2.2	Analysis of Forced Vibration	355
15.3	Free Vibrations of a Rotor	359
15.3.1	State Vector and Transfer Matrix	359
15.3.2	Frequency Equation and the Vibration Mode	364
15.3.3	Examples	365
15.3.3.1	Model I: Uniform Continuous Rotor	365
15.3.3.2	Model II: Disk–Shaft System	366
15.4	Forced Vibrations of a Rotor	367
15.4.1	External Force and Extended Transfer Matrix	367
15.4.2	Steady-State Solution	370
15.4.3	Example	371
	References	371
<b>16</b>	<b>Measurement and Signal Processing</b>	<b>373</b>
16.1	General Considerations	373
16.2	Measurement and Sampling Problem	374
16.2.1	Measurement System and Digital Signal	374
16.2.2	Problems in Signal Processing	375
16.3	Fourier Series	376
16.3.1	Real Fourier Series	376
16.3.2	Complex Fourier Series	376
16.4	Fourier Transform	378
16.5	Discrete Fourier Transform	379
16.6	Fast Fourier Transform	383
16.7	Leakage Error and Countermeasures	383
16.7.1	Leakage Error	383
16.7.2	Countermeasures for Leakage Error	384
16.7.2.1	Window Function	384
16.7.2.2	Prevention of Leakage by Coinciding Periods	385
16.8	Applications of FFT to Rotor Vibrations	386
16.8.1	Spectra of Steady-State Vibration	386
16.8.1.1	Subharmonic Resonance of Order 1/2 of a Forward Whirling Mode	386
16.8.1.2	Combination Resonance	388
16.8.2	Nonstationary Vibration	388
	References	391
<b>17</b>	<b>Active Magnetic Bearing</b>	<b>393</b>
17.1	General Considerations	393
17.2	Magnetic Levitation and Earnshaw's Theorem	393
17.3	Active Magnetic Levitation	394
17.3.1	Levitation Model	394
17.3.2	Current Control with PD-Control	396
17.3.2.1	Physical Meanings of PD Control	397
17.3.2.2	Transfer Function and Stability Condition	397



17.3.2.3	Determination of Gains	398
17.3.2.4	Case with a Static Load	399
17.3.3	Current Control with PID-Control	399
17.3.3.1	Transfer Function and Stability Condition	399
17.3.3.2	Determination of Gains	400
17.3.3.3	Case with a Static Load	400
17.3.4	Practical Examples of Levitation	401
17.3.4.1	Identification of System Parameters	401
17.3.4.2	Digital PD-Control with DSP	402
17.3.5	Current Control with State Feedback Control	403
17.4	Active Magnetic Bearing	405
17.4.1	Principle of an Active Magnetic Bearing	405
17.4.2	Active Magnetic Bearings in a High-Speed Spindle System	405
17.4.3	Dynamics of a Rigid Rotor system	406
	References	408

## **Appendix A Moment of Inertia and Equations of Motion 409**

## **Appendix B Stability above the Major Critical Speed 413**

## **Appendix C Derivation of Equations of Motion of a 4 DOF Rotor System by Using Euler Angles 415**

## **Appendix D Asymmetrical Shaft and Asymmetrical Rotor with Four Degrees of Freedom 421**

D.1	4 DOF Asymmetrical Shaft System	421
D.2	4 DOF Asymmetrical Rotor System	423
	Reference	425

## **Appendix E Transformation of Equations of Motion to Normal Coordinates: 4 DOF Rotor System 427**

E.1	Transformation of Equations of Motion to Normal Coordinates	427
E.2	Nonlinear Terms	428
	References	429

## **Appendix F Routh–Hurwitz Criteria for Complex Expressions 431**

References	432
------------	-----

## **Appendix G FFT Program 433**

References	435
------------	-----

Index	437
-------	-----