

Contents

1	Introduction	1
1.1	Morse Theory and Topography	1
1.2	Mathematical Morphology.....	3
1.3	Inclusion Tree of Level Sets	4
1.4	Topological Description and Computation of Topographic Maps	5
1.5	Organization of These Notes	6
2	The Tree of Shapes of an Image	9
2.1	Introduction and Motivation	9
2.2	Some Topological Preliminaries	11
2.3	The Tree of Shapes.....	18
2.3.1	Tree Structure of the Set of Shapes	19
2.3.2	Branches, Monotone Sections and Leaves	20
2.4	Reconstruction of the Image From Its Tree	24
2.4.1	Direct Reconstruction	25
2.4.2	Indirect Reconstruction	27
2.5	Finiteness of the Tree for Images with Grains of Minimal Positive Size	28
2.6	Applications	30
2.6.1	A Simple Application: Adaptive Quantization	30
2.6.2	Other Applications	31
2.7	Comparison with Component Tree	33
3	Grain Filters	35
3.1	Introduction	35
3.2	General Results From Mathematical Morphology	36
3.2.1	Level Sets and Contrast Invariance	37
3.2.2	Link Between Set Operator and Contrast Invariant Operators on Images	38
3.3	Extrema Filters.....	39
3.3.1	Definition	39
3.3.2	Preliminary Results	40
3.3.3	Properties	43

3.3.4	Interpretation	46
3.3.5	Composition	46
3.3.6	The Effect of Extrema Filters on the Tree of Shapes	48
3.4	Grain Filter	52
3.4.1	Introduction and Definitions	52
3.4.2	Preliminary Results	54
3.4.3	Properties	58
3.4.4	The Effect of Grain Filter on the Tree of Shapes	64
3.5	Relations with Other Operators of Mathematical Morphology .	65
3.5.1	Relations with Grain Operators	65
3.5.2	Relations with Connected Operators	66
3.5.3	Interpretation	70
3.6	Experiments	70
3.6.1	Algorithm	70
3.6.2	Experimental Results	70
3.6.3	Complexity	73
4	A Topological Description of the Topographic Map	75
4.1	Monotone Sections and Singular Values	75
4.2	Weakly Oscillating Functions, Their Intervals and the Structure of Their Topographic Map	81
4.3	Signature and Critical Values	91
4.4	Critical Versus Singular Values	95
4.5	Singular Values of the Tree Versus Singular Values	97
4.6	Review on the Topographic Description of Images	100
5	Merging the Component Trees	103
5.1	The Structure of the Trees of Connected Components of Upper and Lower Level Sets	103
5.2	Construction of the Tree of Shapes by Fusion of Upper and Lower Trees	106
5.3	Fusion Algorithm in the Discrete Digital Case	112
6	Computation of the Tree of Shapes of a Digital Image	115
6.1	Continuous Interpretation of a Digital Image	115
6.1.1	From Digital to Continuous Image	115
6.1.2	Digital Shapes	116
6.2	Fast Level Set Transform	118
6.2.1	Input and Output	118
6.2.2	Description of the Algorithm	119
6.2.3	Justification of the Algorithm	128
6.2.4	Complexity	131
6.2.5	Comparison to Component Tree Extraction	134
6.3	Taking Advantage of the Tree Structure	134
6.3.1	Storage of Pixels of the Digital Shapes	135
6.3.2	Computation of Additive Shape Characteristics	135

6.4	Alternative Algorithms	137
6.4.1	Changing Connectivity	137
6.4.2	Level Lines Extraction	139
6.4.3	Higher Dimension	140
7	Computation of the Tree of Bilinear Level Lines	141
7.1	Level Lines and Bilinear Interpolation.....	141
7.2	Tree of Bilinear Level Lines	145
7.3	Algorithms for the Extraction of the Tree of Bilinear Level Lines (TBLL)	147
7.3.1	Direct Algorithm	148
7.3.2	Morse Algorithm	150
8	Applications	155
8.1	Image Filters	155
8.2	Image Registration	156
8.2.1	Shapes as Features and Their Descriptors.....	157
8.2.2	Meaningful Matches	158
8.2.3	Grouping Matches	159
8.2.4	Eliminating Outliers by Vote	160
8.2.5	Examples.....	161
8.3	Other Applications	164
8.3.1	Image Intersection.....	164
8.3.2	Meaningful Edges	164
8.3.3	Corner Detection	165
8.3.4	Scale Adaptive Neighborhoods	167
8.4	Maximally Stable Extremal Regions.....	167
Conclusion		173
Summary		173
Extensions		173
Self-Dual Filters		174
Fast Level Set Transform in Dimension 3		174
Color Images.....		175
Glossary		177
References		179
Index		185