## **Contents**

1	Logit	Models for Spatial Interaction: Background	1
	1.1	Introduction	1
	1.2	Cost-Minimizing Behavior	3
	1.3	Intuitive Gravity Models and Most Probable State Approach	4
	1.4	User Equilibrium in a Network	6
	1.5	Econometric Models of Probabilistic Choice	6
	1.6	Luce's Axiomatic Derivation	7
	1.7	ARUM – Additive Random Utility Maximization – Approach	7
	1.8	Structured or Nested Logit Models	8
	1.9	Transportation Problem in Linear Programming	8
	1.10	Lagrangian Methods of Deriving Logit Models	9
	1.11	Welfare Measures	10
2	<b>Empi</b> 2.1	rical and Policy Relevance of the New Paradigm	11 11
	2.2	Policy Relevance	12
3	Beha	vioral Foundations of Spatial Interaction Models	15
	3.1	Basic Ideas – Cost-Minimizing Behavior and Equilibrium	16
	3.2	Probability Models	18
	3.3	Freedom of Choice	20
	3.4	Cost-Minimizing Behavior	21
	3.5	The Simple (Multinomial) Logit Model Exhibits	
		Cost-Minimizing Behavior	22
	3.6	Cost-Minimizing Behavior Implies the Logit Model	23
	3.7	Welfare Measure	24
	3.8	Graphical Test	24
	30		25
		Some Particular Discrete Choice Models	25
	3.10	Some Particular Discrete Choice Models	25 25
	3.10 3.11	Some Particular Discrete Choice Models Equilibrium Choice of Origin, Destination and Route	23 25 27
	3.10 3.11 3.12	Some Particular Discrete Choice Models Equilibrium Choice of Origin, Destination and Route Choice of Origin, Destination, Mode and Route	23 25 27 29



Contents	\$
----------	----

3.13 3.14 3.15	Comments Notes About Notation	29 29 30
Part I C	Cost-Minimizing Behavior: Constant Link Costs	
4 Logit	Models for Discrete Choice	33

-			
	4.1	Preliminaries	33
	4.2	The Simple (Multinomial) Logit Model	35
		4.2.1 Formal Derivation of the Simple (Multinomial)	
		Logit Model	36
	4.3	The General Logit Model for Cost-Minimizing Behavior	41
	4.4	Axiomatic Derivations of Logit Models	46
		4.4.1 Axioms for Cost-Minimizing Behavior	47
		4.4.2 Axioms for Payoff-Maximizing Behavior	48
	4.5	Axioms for ARUM Derivation	49
		4.5.1 ARUM Derivation of the Simple (Multinomial)	
		Logit Model	49
		4.5.2 Properties of the Expected Achieved Perceived Utility	51
		4.5.3 Generalized Extreme Value Model	52
	4.6	Extensions	53
		4.6.1 Comments on the Cost Function	53
		4.6.2 Different Interpretations of the Same Model	54
		4.6.3 Cost-Minimizing Behavior for one Decision	
		Maker Making N Repeated Decisions	55
	4.7	Comments	57
	4.8	Notes	60
5	Some	Particular Logit Models	63
	5.1	Introduction	63
	5.2	Stochastic Route Choice	63
	5.3	The Multi-Attribute Discrete Choice Model	64
	5.4	Generalized Cost	65
	5.5	The Gravity Model for Trip Distribution	66
	5.6	The Gravity Model for Trip Distribution with Several	
		Cost Attributes	69
	5.7	Structured (Nested) Logit Models	71
		5.7.1 The Structured Logit Model: The Joint Logit Model	73
		5.7.2 The Standard Nested Logit Model	77
		5.7.3 The Standard ARUM Nested Approach	81
	5.8	The Logit Model with Individual Cost Values	81
	5.9	Socioeconomic Factors	82
	5.10	Comments and Extensions	84
	5.11	Notes	86

x

6	Welf	are, Benefit and Freedom of Choice	87
	6.1	Introduction	. 87
	6.2	Achievement Measure	88
	6.3	Freedom of Choice: Preliminaries	89
	6.4	Freedom of Choice Measure	91
		6.4.1 Freedom of Choice in the Probabilistic Case	93
	6.5	Welfare Measures	94
		6.5.1 Welfare Measure for the Simple Logit Model	94
		6.5.2 Numerical Illustrations	97
		6.5.3 Welfare Measure for the General Logit Model	99
		6.5.4 Welfare Measures for some Particular Models	100
		6.5.5 Welfare Measure for the Stochastic Route Choice Model	100
		6.5.6 Welfare Measure for the Multi-Attribute Case	101
		6.5.7 Welfare Measure for Structured (Nested) Logit Models	101
		6.5.8 Welfare Measure for Gravity Model for Trip Distribution	103
		6.5.9 Welfare Measures for Models	
		with Socioeconomic Factors	103
	6.6	Extensions	104
		6.6.1 Extended Benefit Measure	104
		6.6.2 A Lower Bound on Observed Negative Entropy	105
		6.6.3 Value of Time	106
	6.7	Comments	106
		6.7.1 Revealed Freedom of Choice, Diversity	
		of Choice, Flexibility of Choice	106
		6.7.2 Freedom of choice: Advantage and Achievement	106
		6.7.3 Entropy and Freedom of Choice	107
		6.7.4 Welfare and Benefit Measures	108
	6.8	Notes	108
7	Gran	phical Tests of Cost-Minimizing Behavior	
	in Lo	git Models	109
	7.1	Introduction	109
	7.2	Testing for Cost-Minimizing Behavior in the Simple	
		Logit Model	110
		7.2.1 Graphical Test of Cost-Minimizing Behavior	112
		7.2.2 Asymptotic Distribution of the Observed	
		Negative Entrony	112
		7.2.3 Simulation Experiments	114
		724 Figures	115
		725 Final Remarks	117
	7.3	Multi-Attribute Discrete Choice Models	117
	,	7 3 1 Graphical Test of Cost-Minimizing Behavior	118
	74	Structured Logit Models	119
	,	7.4.1 Graphical Test of Cost-Minimizing Behavior	121
	75	Comments and Extensions	121
	7.6	Notes	122
		······································	

## Part II Equilibrium

8	Equilibrium		125
	8.1	Introduction	125
	8.2	Smith's Cumulative Cost Function and Equilibrium	125
	8.3	Route Choice	127
		8.3.1 Continuous Approximation	129
	8.4	Choice of Origin, Destination and Route	130
		8.4.1 Most Probable Trip Pattern	131
		8.4.2 Most Probable Flow	133
		8.4.3 Continuous Approximation	134
	8.5	Choice of Origin, Destination, Mode and Route	137
		8.5.1 Continuous Approximation	139
	8.6	Comments and Extensions	140
	8.7	Notes	141
9	App	endix	143
	9.1	Representation Theorem for Cost-Minimizing	
		Probability Distributions	143
	9.2	Likelihood and Entropy of a Sample	145
	9.3	Maximum Entropy	146
	9.4	Maximum Likelihood	147
Re	feren	ces	149